Routing Games : From Altruism to Egoism

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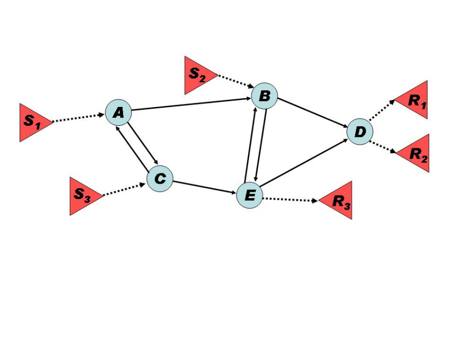
Outline

Routing

General Routing



- Input
  - network topology, link metrics, and traffic matrix
- Output
  - set of routes to carry traffic

Network Routing : Classical Approach

- Routing as optimization problem
  - e.g., minimum total delay in network
  - focus on global network performance (social optimal)
  - performance of individual user not important
- Centralized or distributed algorithms
  - e.g., link state or distance vector

Network Routing : Game-Theoretic Approach

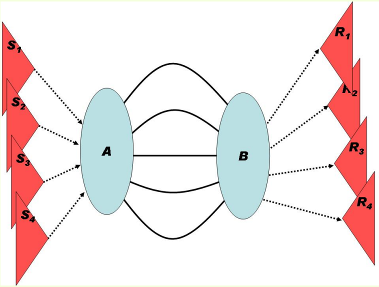
- Routing as game between users
  - users determine route
  - decision based solely on individual performance (selfish routing)
  - strongly dependent on other users decisions
- Non-cooperative game (non-zero sum)
  - users compete for network resources
- Equilibrium point of operation
  - Nash equilibrium point (NEP)

More

- Competitive routing in multiuser communication networks
  A. Orda, R. Rom and N. Shimkin
  IEEE/ACM Transactions on Networking, 1 (5) 1993
- How bad is selfish routing ?
  T. Roughgarden and E. Tardos
  Journal of the ACM, 49 (2) 2002
- Selfish routing with atomic players
  T. Roughgarden
  ACM/SIAM Symp. on Discrete Algorithms (SODA) 2005

Simple Model : Network of Parallel Links

Parallel Links



- set of users share a set of parallel links
- each user has fixed demand (data rate)
- users decide how to split demand across links
  - minimize individual cost
- link has a load dependent cost (e.g., delay)

System Model

- Network : a graph $\mathcal{G} = (\mathcal{V}, \mathcal{L})$
  - $\mathcal{V}$ is a set of nodes
  - $\mathcal{L} \subseteq \mathcal{V} \times \mathcal{V}$ is set of directed links.
- $\mathcal{I} = \{1, 2, ..., I\}$ is a set of users which share the network $\mathcal{G}$.
- f_l^i= flow of user i in link l .
- Each user i has a throughput demand rate r^i (which can be split among various path).
- Strategy : $\mathbf{f}^i = (f_l^i)_{l \in \mathcal{L}}$ is the routing strategy of user i.

Assumptions :

- At least one link exist between each pair of nodes(in each direction).
- Flow is preserved at all nodes.

Nash Equilibrium

- Cost/Utility function $J^i(\mathbf{f}) = \sum_l f_l^i \mathcal{T}_l(f_l)$.

Each user seeks to minimize the cost function J^i, which depends upon routing strategy of user i as well as on the routing strategy of other users.

Nash Equilibrium

A vector $\tilde{\mathbf{f}}^i$, $i = 1, 2, ..., I$ is called a Nash equilibrium if for each user i, $\tilde{\mathbf{f}}^i$ minimizes the cost function given that other users' routing decisions are $\tilde{\mathbf{f}}^j$, $j \neq i$. In other words,

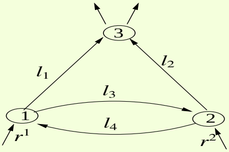
$$\tilde{J}^i(\tilde{\mathbf{f}}^1, \tilde{\mathbf{f}}^2, ..., \tilde{\mathbf{f}}^I) = \min_{\mathbf{f}^i \in \mathbf{F}^i} \hat{J}^i(\tilde{\mathbf{f}}^1, \tilde{\mathbf{f}}^2, ..., \mathbf{f}^i, ..., \tilde{\mathbf{f}}^I), \quad i = 1, 2, ..., I \quad , \tag{1}$$

where $\mathbf{F}^i$ is the routing strategy space of user i.

Network Topology

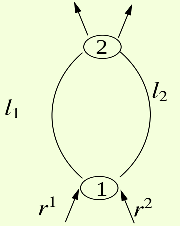
Consider the following network topology

Load Balancing Network



$$\hat{J}^i = \sum_{l \in \{1,\ldots 4\}} f_l^i T_l(f_l)$$

Parallel Link Network



$$\hat{J}^i = \sum_{l \in \{1,2\}} f_l^i T_l(f_l)$$

Cost Function

Consider the following Cost function.

Linear Cost Function

- Used in Transportation Networks
- $T_l(f_{l_i}) = a_i f_{l_i} + g_i$ for link $i = 1, 2$, where as, $T_l(f_{l_j}) = cf_{l_j} + d$ for link $j = 3, 4$.

M/M/1 Delay Cost Function

- Used in Queueing Networks
- $T_l(f_{l_i}) = \frac{1}{C_{l_i} - f_{l_i}}$, where the C_{l_i} and f_{l_i} denote the total capacity and total flow of the link l_i.

For parallel link topology only link l_i, $i = 1, 2$ exist while for load balancing topology link l_i, $i = 3, 4$ also exist.

Related work

For Selfish Users

Orda et al

Ariel Orda, Raphael Rom, and Nahum Shimkin, “ Competitive Routing in Multiuser Communication Networks”,*IEEE/ ACM Transactions on Networking*, Vol.1 No. 5, October 1993

Kameda et al

H. Kameda , E. Altman, T. Kozawa, Y. Hosokawa , “Braess-like Paradoxes in Distributed Computer Systems” , *IEEE Transaction on Automatic control*, Vol 45, No 9, pp. 1687-1691, 2000.

- Orda et al has shown unique Nash equilibrium for Parallel link network with MM1 cost function.
- Kameda et al also claim unique Nash equilibrium for Load balancing network with MM1 cost function.
- Braess like paradox is observed by Kameda et al in Load balancing network with MM1 cost function.

What happens with ”User Cooperation” ?

Degree of Cooperation

Definition

Let $\overrightarrow{\alpha^i} = (\alpha^i_1, .., \alpha^i_{|\mathcal{I}|})$ be the *degree of Cooperation* for user i. The new operating cost function $\hat{J}^i$ of user i with Degree of Cooperation, is a convex combination of the cost of user from set $\mathcal{I}$,

$$\hat{J}^i(\mathbf{f}) = \sum_{k \in \mathcal{I}} \alpha^i_k J^k(\mathbf{f}); \ \sum_k \alpha^i_k = 1, i = 1, ...|\mathcal{I}|$$

- Non cooperative user : $\alpha^i_k = 0 \text{ for all } k \neq i \Rightarrow$ User i takes into account of only its cost
- Cooperative (Equally cooperative) : $\alpha^i_j = \frac{1}{|\mathcal{P}|}$, where, $j \in \mathcal{P}, \mathcal{P} \subseteq \mathcal{I} \Rightarrow$ User i takes into account the cost of each users j(including itself).
- Beyond Cooperation - Altruistic user : $\alpha^i_i = 0 \Rightarrow$ User i takes into account the cost of only other users

With Cooperation

Each user still seeks to minimize the operating cost function $\hat{J}^i$.

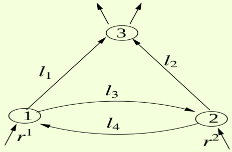
Non-Cooperative Framework

We can benefit to apply the properties of non-cooperative games. e.g. (Nash Equilibrium etc.)

Network Topology

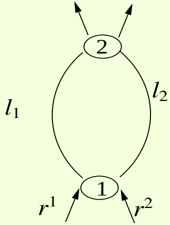
Consider the following network topology

Load Balancing Network



$$\hat{J}^i = \sum_{l \in \{1,\ldots 4\}} \sum_{k \in \{1,2\}} \alpha^i_k f^k_l T_l(f_l)$$

Parallel Link Network



$$\hat{J}^i = \sum_{l \in \{1,2\}} \sum_{k \in \{1,2\}} \alpha^i_k f^k_l T_l(f_l)$$

Related work

On Various degree of Cooperation

Michiardi Pietro, Molva Refik A game theoretical approach to evaluate cooperation enforcement mechanisms in mobile ad hoc networks WiOpt'03

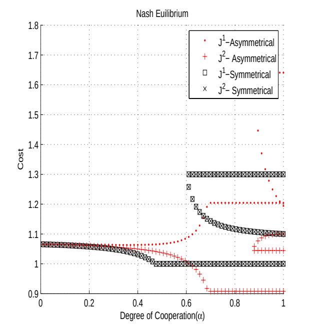
On Altruism

Handbook of the Economics of Giving, Altruism and Reciprocity, Volume 1, 2006, Edited by Serge-Christophe Kolm and Jean Mercier Ythier

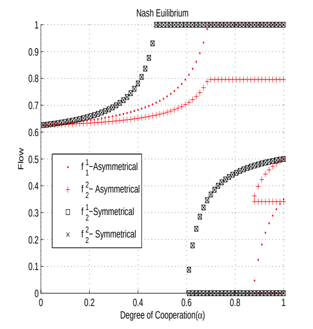
”Motivationally, altruism is the desire to enhance the welfare of others at a net welfare loss to oneself.”

Load Balancing Network with Linear link Cost

Cost at Nash Equilibria



Flow at Nash Equilibria



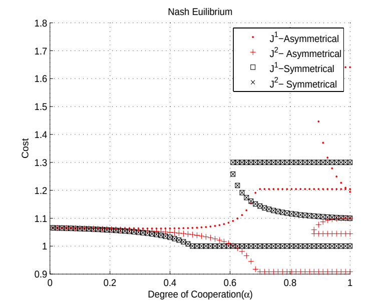
Parameters : $a = 1, c = 0, d = 0.5$,
Cooperation : { Symmetrical : $\alpha^1 = \alpha^2$, Asymmetrical : $0 \leq \alpha^1 \leq 1, \alpha^2 = 1$}

Some strange observation

- Multiple Nash equilibrium ...

Cooperation Paradox

Cost at Nash Equilibrium



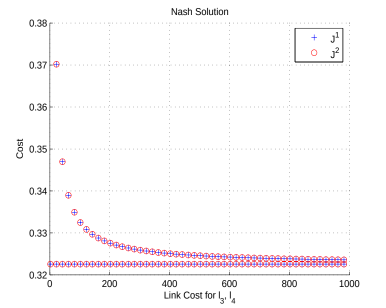
Parameters : $a = 1, c = 0, d = 0.5$.

Cooperation Paradox : Cooperation improves the cost.

- Selfishness is not good always :)

Braess like Paradox

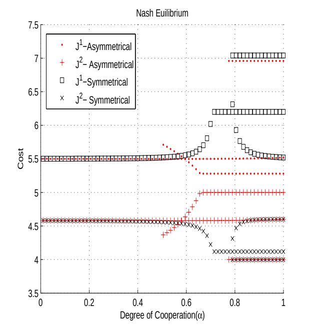
Cost at Nash Equilibrium



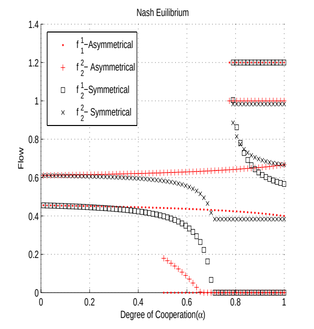
Parameters : $a_1 = a_2 = 4.1, d = 0.5,$
Symmetrical : $\alpha^1 = \alpha^2 = 0.93$
Braess Paradox : Additional resources degrades the performance.

Parallel Link Network with Linear link Cost

Cost at Nash Equilibrium



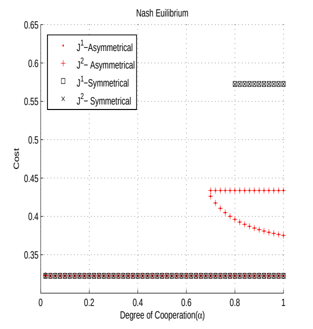
Flow at Nash Equilibrium



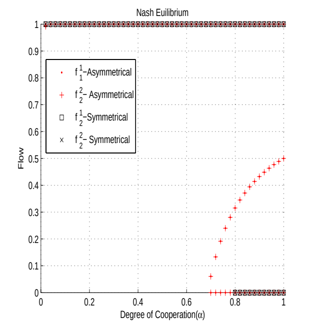
Parameters : $a = 1, c = 0, d = 0.5.$

Load balancing network with M/M/1 link cost

Cost at Nash Equilibrium



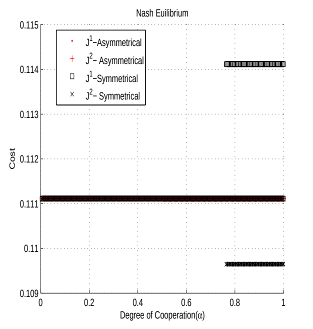
Flow at Nash Equilibrium



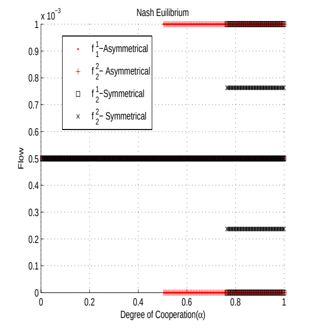
Parameters : $a = 1, c = 0, d = 0.5$.

Parallel link with M/M/1 link cost

Cost at Nash Equilibrium



Flow at Nash Equilibrium



Parameters : $a = 1, c = 0, d = 0.5$.

Observation Summary

- Uniqueness of NEP is lost
- Paradox in Cooperation
- Braess like paradox

Assumptions on Cost function

Consider the following assumption on the Cost function J^i

Type G function- Assumptions

G1 : $J^i(\mathbf{f}) = \sum_{l \in \mathcal{L}} \hat{J}^i_l(f_l))$. Each $\hat{J}^i_l$ satisfies :

G2 : $J^i_l : [0, \infty) \rightarrow (0, \infty]$ is continuous function.

G3 : J^i_l : is convex in f^j_l for $j = 1, ... |\mathcal{I}|$.

G4 : Wherever finite, J^i_l is continuously differentiable in f^i_l, denote $K^i_l = \frac{\delta \hat{J}^i_l}{\delta f^i_l}$.

Existence of NEP is shown to exist in Orda et al for Selfish users operating on parallel link.

More

Existence and Uniqueness of NEP

Cost functions

$$\hat{J}_l^i(\mathbf{f}) = \sum_{l \in \mathcal{L}} (\alpha^i f_l^i + (1-\alpha^i) f_l^{-i}) T_l(f_l)$$
$$= \sum_{l \in \mathcal{L}} (\alpha^i f_l + (1-2\alpha^i) f_l^{-i}) T_l(f_l)$$

Existence can be studied as in Orda et al. (Shown to exist.)

Uniqueness of NEP

- for $\alpha^i \leq 0.5$ - Unique - Extended from Orda et al
- for $\alpha^i > 0.5$ - Not Unique (Because $K_l^i(f_l^{-i}, f_l)$ is not strictly increasing function in f_l^{-i} and f_l).

Uniqueness of NEP

Still some unique NEP can be obtained for ($\alpha > 0.5$)

Theorem

Consider the cost function of type B. Let $\hat{\mathbf{f}}$ and $\mathbf{f}$ be two Nash equilibria such that there exists a set of links $\overline{\mathcal{L}}_1$ such that $\{f_l^i > 0$ and $\hat{f}_l^i, i \in \mathcal{I}\}$ for $l \in \overline{\mathcal{L}}_1$, and $\{f_l^i = \hat{f}_l^i = 0, i \in \mathcal{I}\}$ for $l \notin \overline{\mathcal{L}}_1$. Then $\hat{\mathbf{f}} = \mathbf{f}$.

Unique NEP can be seen for some α.

Mixed Equilibrium

Network is shared by two types of users :

a. *group users* : have to route a large amount of jobs ; Seek Wardrop eqiilibria.
b. *individual users* : have a single job to route ; Seek Nash equilibria.

Studied by Harker (88), Eitan et al (2000).

- Unique equilibria with M/M/1 cost function.

Mixed Equilibrium

Cost function

- $J^i : \mathbf{F} \rightarrow [0, \infty)$ is the cost function for each user $i \in \mathcal{N}$.
- $\mathcal{F}_p : \mathbf{F} \rightarrow [0, \infty)$, is the cost function of path p for each individual user.

The aim of each user is to minimize its cost, i.e., for $i \in \mathcal{N}$, $\min_{f^i} J^i(\mathbf{f})$ and for individual user, $\min_{p \in \mathcal{P}} \mathcal{F}^i_p(\mathbf{f})$. Let fp be the amount of individual users that choose path p.

Definition

$\mathbf{f} \in \mathbf{F}$ is a Mixed Equilibrium (M.E.) if

$$\forall i \in \mathcal{N}, \forall g^i \text{s.t.} (\mathbf{f}^{-i}, g^i) \in \mathbf{F}, \hat{J}^i(\mathbf{f}) \leq \hat{J}^i(\mathbf{f}^{-i}, g^i)$$
$$\forall p \in \mathcal{P}, \mathcal{F}_{(p)}(\mathbf{f}) - A \geq 0;\ (\mathcal{F}_{(p)}(\mathbf{f}) - A) f^i_{(p)} = 0$$

where $A = \min_{p \in \mathcal{P}} \mathcal{F}_p(\mathbf{f})$

Mixed Equilibrium with Cooperation

We obtain closed form solutions with cooperation (α) for a parallel link network with M/M/1 cost function.

- *When Both link is used at Wardrop equilibrium :*

$$\begin{cases} (M_1, N_1) & \text{if } a_1 < M_1 < b_1; \\ otherwise, & \\ (0, -cc) & \text{if } r_1 < \min\left(r_2 + C_2 - C_1, \frac{\alpha(C_2 - C_1) + 2\alpha r_2}{2\alpha - 1}\right), \\ (r_1, r_1 - cc) & \text{if } r_1 < \min\left(\frac{\alpha(C_2 - C_1)}{1 - 2\alpha}, r_2 - (C_2 - C_1)\right), \end{cases}$$

where

$$M_1 = \frac{-\alpha(C_2 - C_1) + r_1(2\alpha - 1)}{2(2\alpha - 1)}, \quad N_1 = \frac{(C_1 - C_2)(1 - \alpha) + (2\alpha - 1)r_2}{2(2\alpha - 1)},$$

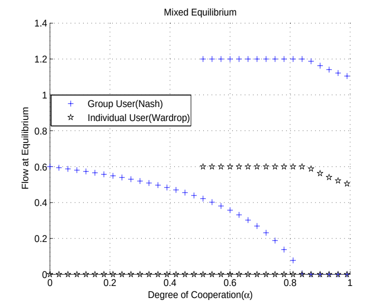
$$a_1 = \max(-\frac{C_2 - C_1}{2} - \frac{r_2 - r_1}{2}, 0), \quad b_1 = \min(-\frac{C_2 - C_1}{2} + \frac{r_1 + r_2}{2}, r_1),$$

$$cc = -\frac{C_2 - C_1}{2} - \frac{r_2 - r_1}{2}, \quad dd = -\frac{C_2 - C_1}{2} + \frac{r_2 + r_1}{2},$$

- When only one link (link 1) is used at Wardrop equilibrium :
- When only one link (link 2) is used at Wardrop equilibrium :

Mixed Equilibrium

Flow at Equilibrium



Parameters : $C_{l_1} = 4, C_{l_2} = 3, r^1 = 1.2, r^2 = 1$

Multiple Equilibria

Concluding Remarks

We parameterize the "degree of Cooperation" to capture the behavior in the regime from altruistic to egocentric and identify some strange behavior

- Loss of uniqueness
- Cooperation paradox - Typically caused due to several equilibria.
- Braess Paradox - Typically caused due inefficiency.

Perspective

Many questions are raised

- How does the system behave when the users cooperate with more fairness , e.g., α fairness ?
- How does the cooperation behaves for an hierarchical routing game (Stackelberg games) ?
- How does the similar routing games behave in dynamic environment ?
- Few more - Measure of inefficiency(e.g., price of anarchy vs price of stability), Selection of desired equilibria, Convergence to desired equilibria.

Thanks

Questions ?

Routing : different methods

Optimization problem :

- single control objective
  eg. optimization of average network delay
- Either centralized or distributed control
- Passive Users

Game theoretic : resource shared by a group of active users

- Each user optimize its own cost/performance
- A non-cooperative game
- Existence, uniqueness, paradoxes ?

Back

Assumptions on Cost function

Type B function- Assumptions

B1 : $J^i(\mathbf{f}) = \sum_{l\in\mathcal{L}} f_l^i T_l(f_l))$

B2 : $T_l : [0,\infty) \rightarrow (0,\infty]$.

B3 : $T_l(f_l)$ is positive, strictly increasing and convex.

B4 : $T_l(f_l)$ is continuously differentiable.

Type C function

C1 : $\hat{J}^i(f_l^i, f_l) = f_l^i T_l(f_l)$ is a type-B cost function.

C2 : $T_l = \begin{cases} \frac{1}{C_l - f_l} & f_l < C_l \\ \infty & f_l > C_l \end{cases}$.

Where C_l is the capacity of the link l.

Note that type C is a special kind of type B function which correspond to M/M/1 delay function.

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